Strategy for optimizing
Sailplane wing aerodynamics
With constraint span

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This paper presents a strategy for dealing with aerodynamic optimization of wing with constraint span (typically 15m/standart glider class, or 60inch model gliders).

Aerodynamic study are performed in term of aerodynamic coefficient. The aerodynamic coefficient is defined with regards to a reference area, which is most of the time the actual wing area.
When optimising for a given span, the degree of freedom is more or less wing area. As a result, when comparing two designs based on classical aerodynamic coefficient, one is comparing sets with different reference area. This leads to difficulties to properly interpret the difference between two designs.
The current method aims at providing a set of aerodynamic coefficient more handy at comparing design with different areas.

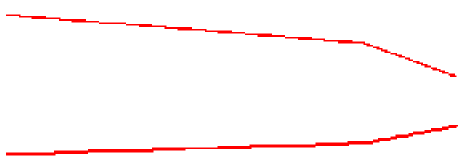
Aerodynamic coefficient for fixed reference area

The key point for comparing wing with different area is to compare aerodynamic coefficient for a given reference area. It means that the aerodynamic coefficient will not be necessarily expressed with reference to the actual area of the wing.

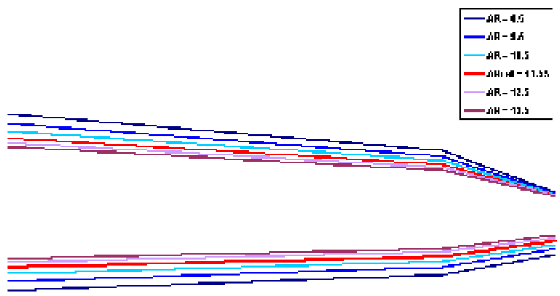
Here are some definition for the different aerodynamic coefficient to be used :

- CL, CD refers to the lift and drag coefficient in the usual meaning, ie with regard to actual wing area S :
  $CL = \frac{L}{Q*S}$ & $CD = \frac{D}{Q*S}$, with Q the dynamic pressure
- CL_{Sref}, CD_{Sref} refers to the lift and drag coefficient with regard to an "arbitrary" reference area :
  Then $CL_{Sref} = \frac{S}{Sref} CL$ & $CD_{Sref} = \frac{S}{Sref} CD$

In order to nevertheless perform a job as comprehensive as possible, it is a good idea to work with a reference wing, which area will be retained as reference area. Then variation of this reference wing will be performed for a given span.



Reference wing



Family of wings for a given span

The optimum area determination will be greatly helped by considering aerodynamic coefficient defined with regards to constant reference area.

If the variation around the reference wing are small enough, the approximation of constant mass over the family of wings can be valid, particularly for small scale A/C as glider or model of glider.

Working for a given CL_{Sref} range wrt constant reference area
means working for a given speed range.

Optimisation for a given CL_{Sref} will be the main bone of proposed method

Drag for constant reference area & given CL_{Sref}

Aerodynamic computations are usually performed with reference to wing area. A set of formula is then needed to transform those results into constant reference area.

What is known is drag polar wrt wing area, given by $CD = CD_S(CL)$. This function is used to defined the drag for constant reference area & given CL_{Sref}, as follows :

$$CD_{Sref}(CL_{Sref}) = \frac{S}{Sref} * CD_S\left(\frac{Sref}{S} CL_{Sref}\right)$$

Case of induced drag

In the case of given planform proportion, the Oswald factor k is kept, and an interesting result on induced drag arises :

As for a given wing, with actual area S, we have $CDi(CL) = \frac{k}{\pi\lambda} CL^2$ with $\lambda = \frac{b^2}{S}$

It comes

$$CD_{Sref}(CL_{Sref}) = \frac{S}{Sref} * \frac{k}{\pi \frac{b^2}{S}} \left(\frac{Sref}{S} CL_{Sref}\right)^2 = \frac{k}{\pi \frac{b^2}{Sref}} (CL_{Sref})^2 = \frac{k}{\pi\lambda ref} (CL_{Sref})^2$$

The induced drag so expressed is independent upon actual area S of the wing !

It means that induced drag of a family of wings with fixed span, homothetic proportion, computed for a given CL_{Sref} (ie speed) has always the same induced drag (in N).

To put it in a nutshell, for given span an increase of Aspect Ratio leads to higher lift coefficient CL to reach one targeted speed, that cancels exactly the geometric effect of AR on Di.

Case of airfoil drag

As induced drag dependency to AR for given span is related to Oswald factor variation only, it means that optimisation is largely ruled by airfoil drag dependency to aspect ratio, hence to chord depth (Reynolds number).

For a given airfoil, drag coefficient CD is given by the function : $CD_{Airf}(CL) = CD_{Airf}(\mathrm{Re}(Chord), CL)$

At first order of magnitude, Chord is given by $Chord = \eta \frac{S}{b}$, with η a form factor

So for Drag values wrt constant reference area & given CL_{Sref}, it comes a function of wing area S, given by : $CD_{Airf,Sref}(CL_{Sref}) = \frac{S}{Sref} CD_{Airf}\left(\mathrm{Re}\left(\eta \frac{S}{b}\right), \frac{Sref}{S} CL_{Sref}\right)$

A decrease of S leads to less wetted area (lower drag values), but to a higher CL & lower chord (higher drag values).

At the end the balance between the different aspect is widely dependant upon the Reynolds range associated to the optimisation : the lower the Re number (eg sailplanes & sailplane models) the higher the dependency of airfoil drag to chord variation

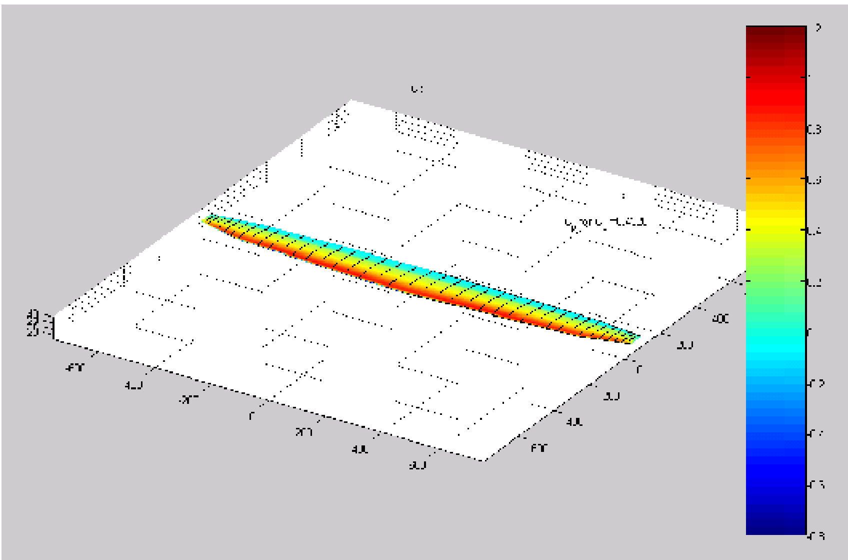
More detailed comparison

The two preceding paragraphs gives simplified views on airfoil & induced drag topic. It nevertheless embodied very well the different physical phenomenon at stake.

For design purpose, more detailed and accurate computations are needed. If there is no constraint on the cord distribution, it will be then possible to optimize planform to reach the best balance between the effects on induced drag (elliptical planform for lower Cdi on planar wings) and airfoil drag (larger chord for lower Cdairfoil).

Such a method is implemented in MIAReX code.

http://pagesperso-orange.fr/scherrer/matthieu/aero/miarex.html

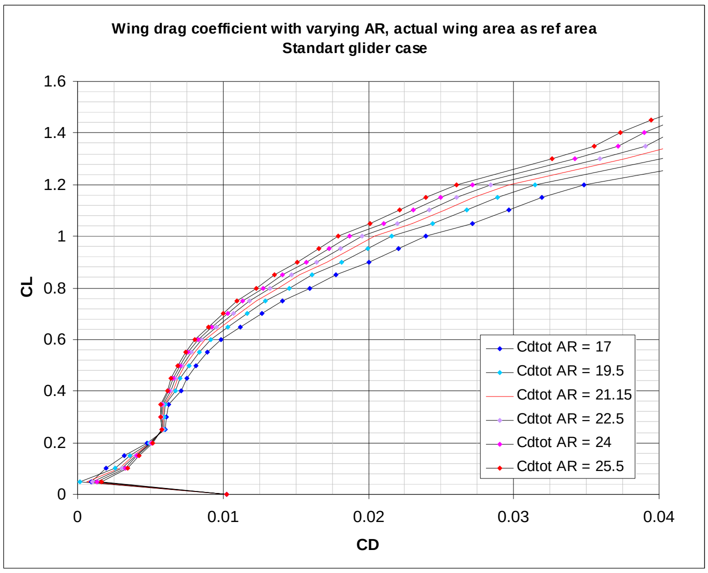


Examples

Some practical cases will illustrate the kind of insight into design process this (twisted…) way to look at aerodynamics coefficient can be useful.

Standart class glider – homothetic planform

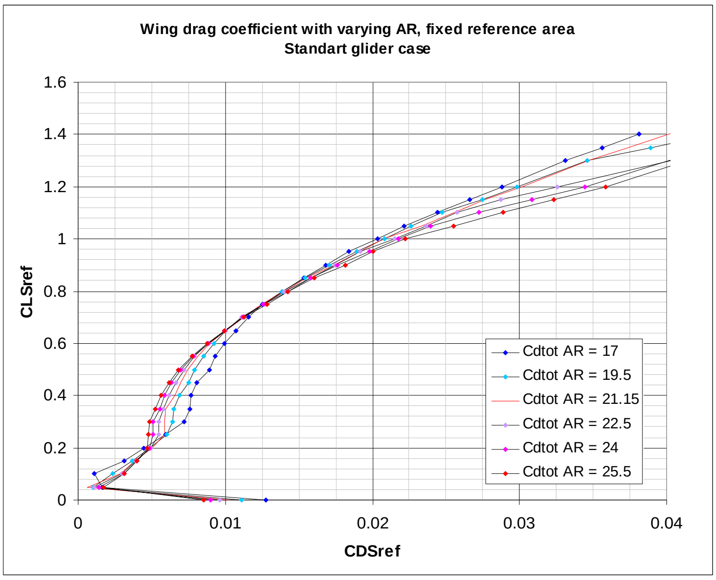
When considering aerodynamic coefficient, here is the usual view on AR effect. It nevertheless does not embody the change of wing area, and we are more or less comparing apples & oranges.



CD(CL)

Usual view : "True" aerodynamic coefficient ref. to actual wing area

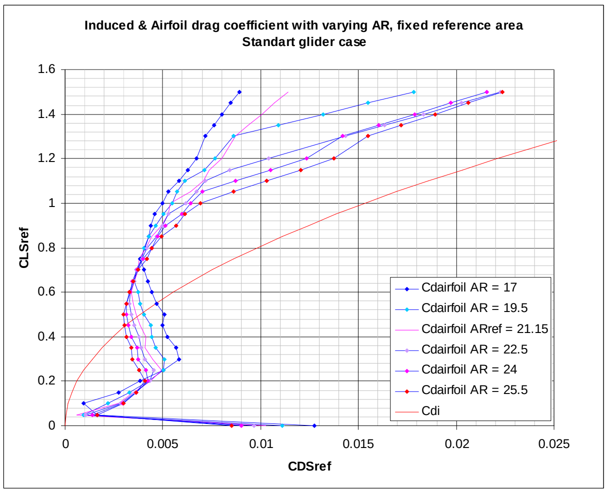
For a given mass, a CL_{Sref} correspond to one same speed on the speed polar for each glider with different wing area. Based on the following plot, it is possible to observe how the aerodynamic performance evolves in comparison to reference wing (pink curves) for a given speed range.
This way of comparison is closer to operational demands when flying sailplanes.



$CD_{Sref}(CL_{Sref})$

Operational view : Aerodynamic coefficient ref. to fixed ref area & for constant CL_{Sref} (ie speed)

What we see here is that higher AR leads to lower CD_{Sref} at low CL_{Sref}, and higher CD_{Sref} at high CL_{Sref}. Well, lower chord would lead to lesser airfoil drag ?
We can have a look more in the detail, and plot CD_{Sref}_airfoil & CD_{Sref}_i.

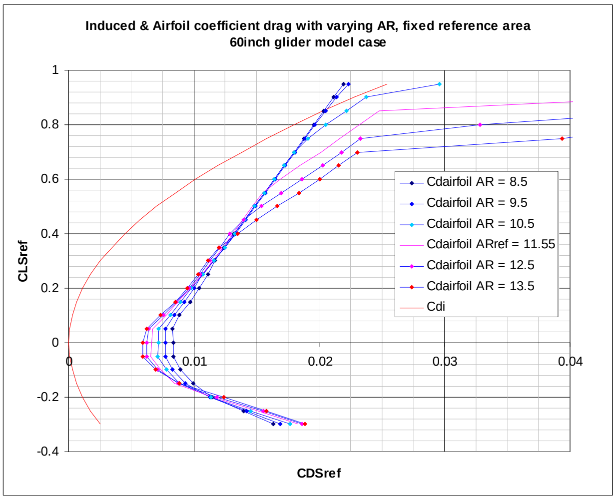


For a given speed, induced CD_{Sref} is virtually constant over the different homothetic wings (already detailed) : then very dependency to AR / area comes from the airfoil drag.
The conclusion drawn here means that a driving factor when considering speed polar is related to area : less area of wing leads to less airfoil drag at low CL_{Sref} thanks to less wetted area.

When speaking of given observable speeds in flight, "true" aerodynamics effects in the (usual) wing-area-reference are much altered by area effect.

60 inch glider model – homothetic planform

Similar trend is observed for 60 inch class glider model : high AR seems to favour high speed, but rapidly increase drag for lower speed, preventing to fly efficiently there.



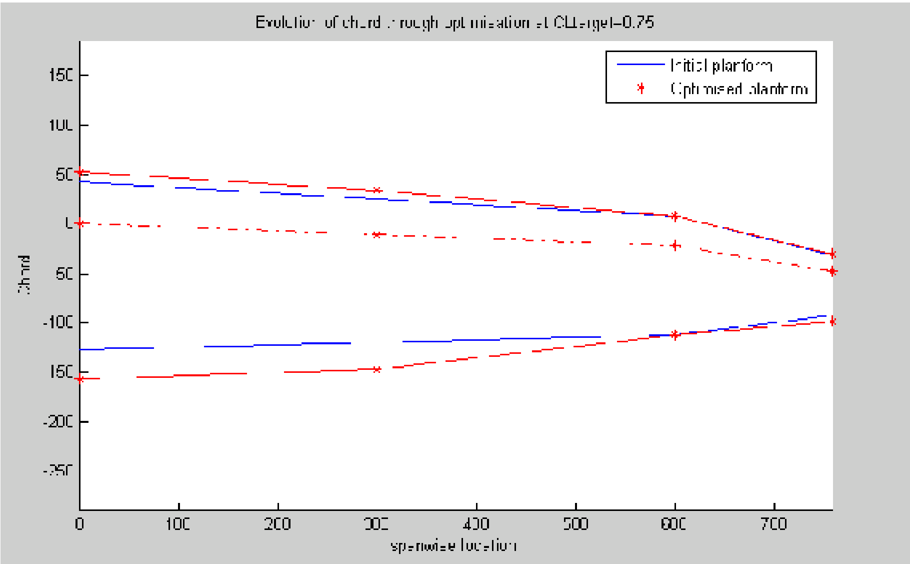
Nevertheless a notable fact is related to the lower Reynolds number band [100 000 – 500 000] the airfoil is functioning in : the variation of airfoil drag with chord (in "true" aerodynamic coefficient) is much steeper that for full size standard class glider.
As a result, "true" aerodynamics plays a bigger role : when considering lower AR, the increase of area does not hurt as much, because the increase of chord enhance the airfoil functioning greatly even for small Re number variation.

60 inch glider model – multi DOF optimisation

More detailed view, taking several airfoil characteristics along span into consideration, allows more specific optimization of wing planform. The impact of changes of any chord on either local airfoil drag, wing AR and Oswald factor can be optimized.

Here is the result of automated optimization implementing the idea in this paper. Four chords are set by the optimizer to minimize CD_{Sref} for one CL_{Sref}. Plan form is changed accordingly, and exchange between induced drag and Airfoil drag can be studied.

	CD_{Sref} tot	CD_{Sref} i	CD_{Sref} Airf	L/D
Initial value	0.034774	0.016068	0.018707	21.5676
Final value	0.032742	0.015911	0.01683	22.9067
Progress made	-5.85%	-0.97%	-10.03%	6.21%



Conclusion

The $CD_{Sref}(CL_{Sref})$ is not a curve involving “true” aerodynamic coefficient in the common sense, but is a practical way to take into account wing area variation in span constraint aerodynamic optimisation. It must be used as an indicator allowing to study the net impact of change in wing area on the practical performance over the speed polar, to be balanced with actual aerodynamic effects.

It is also a *necessary* tool in building cost function for span-constraint automated aerodynamic optimization.