Extended longitudinal stability theory at low Re
-
Application to sailplane models

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November 2006

C_L	Lift coefficient
C_m	Pitching moment coefficient
$C_m{}^{25}{}_W$	Pitching moment coefficient of the wing at 25% of AMC
$X^{\alpha}{}_{NP\ W}$	Neutral point with respect to incidence
$X^{V}{}_{NP\ W}$	Neutral point with respect to V

1. Defects of classical longitudinal theory for low Re case

It appears that the usual linearized theory that you can develop “by the books” has some difficulties for representing the dynamic behaviour and all the stability issues encountered at low Reynolds by sailplane models.
This subject is of importance particularly for sailplane models, because stability is a key factor for **handling and response to the atmospheric movements**, thus “operational performance” at the end.

The usual linearized method often features **de-coupling hypothesis** between short period mode and phugoïd that seems to be doubt full in the case of sailplane models.
This linearized theory also defined a single point of importance for stability issues, the “neutral point”. It is a function of wing and tail geometry, and airfoil lift slope. The neutral point should then be used for defining everything about stability.

1.1. Illustration of non explained phenomenon

Here are some illustrations that usual handling quality “by the books” theories are not sufficient for explaining what is happening at low Re for sailplane models in flight.

While setting the CG for there sailplane modellers do sometime use the so called “diving test” method.
It consists in trimming the sailplane for level flight first. Then a short input of down elevator is created in order to enter the sailplane into a moderate diving. The modeller observes what the behaviour of the sailplane is during the following seconds.

- If the sailplane enter a phugoïd, the CG is set too much forward.
- If the sailplane features a diverging diving, the CG is set too aft.
- If the sailplane keep a rather straight, indifferent path, then the CG would be OK.

Interpreted in term of usual uncoupled & linearized theory, this corresponds to observing the phugoïd behaviour. Whereas this uncoupled theory does predict that phugoïd is independent to CG location. Here is a first paradox between this simplified theory and what modeller do experience in flight.

Another illustration to this is the fact that among modeller common wisdom some airfoil are well knows as “difficult to properly set the CG” and other “easy to set the CG”. Whereas the usual theory does not predict that one or the other airfoil should cause difficulty to find the right CG in the usual flight range..
This aspect even if it is a very subjective topic to decide whether the CG is “easy to set”, should rely on some physical reasons.

1.2. Causes for non representativeness

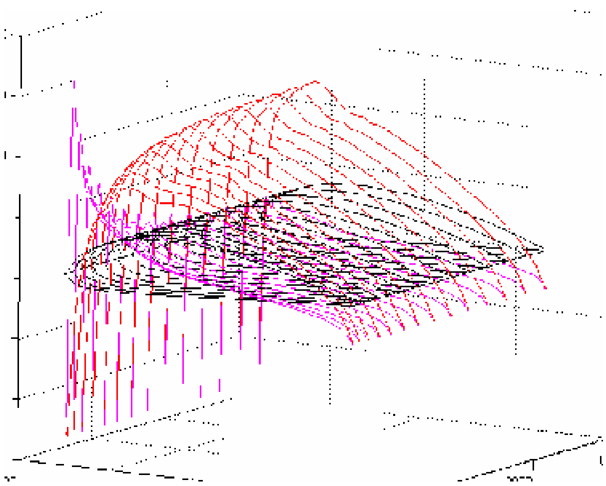
The theory was usually developed in the book for full size aircraft studies. The main point is that model of sailplane do feature specific aerodynamic specificities, that create more complex dynamic behaviour.

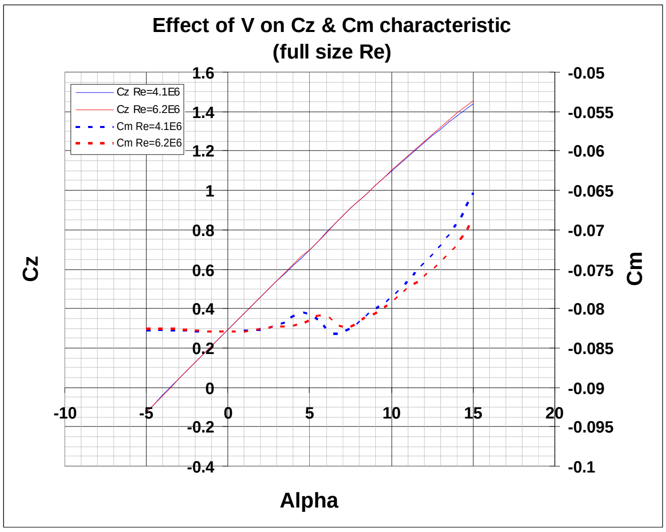
The usual linear theory features following modelling for lift and pitching moment coefficients:

$$C_L = C_{L\alpha} * (\alpha - \alpha_0)$$

$$C_m = C_m0 + C_{m\alpha} * \alpha$$

This linear representation of the physics is reasonably valid for high Reynolds number, above 2 000 000, meaning typically for full size aircraft.





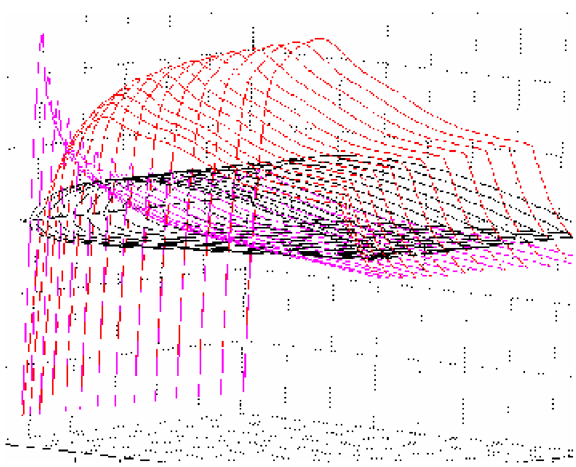
No effect of V on C_L & moderate effect on C_m

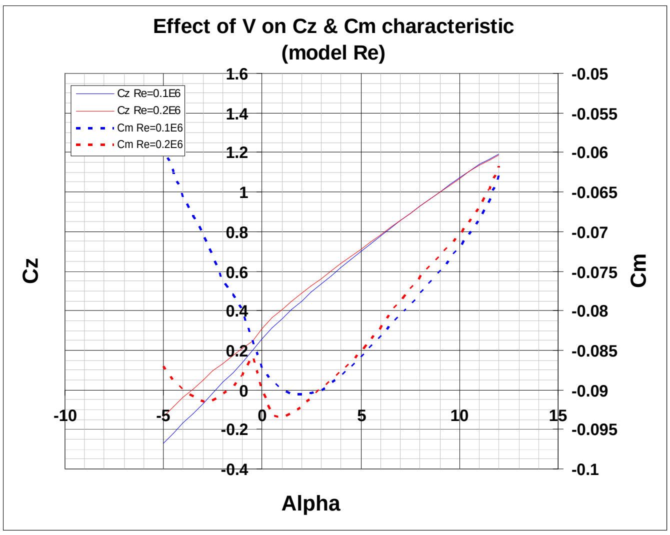
Illustration : For a rectangular wing at Re= 4 000 000 & 6 000 000, the laminar bubble is very reduced, and dependence of aerodynamic behaviour to velocity (Re number) is small.

As soon as the Reynolds number do reach medium values, as encountered for sailplane models (Re within range [80 000 -> 600 000]), this representation is not anymore valid. C_L and C_m are function both of angle of attack α and velocity V, which can be widely non linear.

$$C_L = C_L(\alpha, V)$$

$$C_m = C_m(\alpha, V)$$





Major effect of V on C_L & C_m

Illustration : For a rectangular wing at Re= 100 000 & 200 000, the laminar bubble is clearly visible, and dependence of aerodynamic behaviour to velocity (Re number) is big.

This kind of non linear behaviour do require specific treatment for doing a usable stability analysis.

The scope of the following theory is to extend the concept of “neutral point”, which is usually valid whatever speed V and lift coefficient C_L.
By studying each point of the entire C_L and speed range of a sailplane, it is possible to define the excursion of a “local neutral point”, that defines locally the stability behaviour of the sailplane for this point (C_L, V).
At the end, it is possible to define ***a variable stability over the flight range***, and to have some details about its evolution.

NB : The variable stability experienced within the flight envelope justify the need for a “static ***margin***”.

- For full size aircraft, non linear behaviour occurs mainly near the stall, and is not very dependant on V. The margin is sufficient for covering main aspect about longitudinal stability, and is defined for securing the stall behaviour.
- For models, non linear behaviour occurs within the usual flight range, and is very dependant of V. The concept of “margin” is here not sufficient, and some detailed analysis is necessary for properly studying stability and handling behaviour.

2. Locally linearized theory

2.1. Neutral point with respect to α

Definition :
The neutral point with respect to α is the point were an increment (i.e. a perturbation) of angle of attack α does not generate any pitching moment.

NB : The neutral point with respect to α is the "usual neutral point" we can find in the books.

2.1.1. Usually admitted theory

Pitching moment is usually modelled as :

$$C_m = C_m^{25}{}_W + V_H\, Cz_{\alpha H}(1\text{-}d\varepsilon/d\alpha)*\alpha$$

- Considering **linearized theory** results in the fact hat this point is fixed over the CL range. It is a function of tail vs. wing design (tail volume VH), and of lift slope for both tail and wing characteristics, trough the relation

  $$X_{NP} = X_{NP\,W} + V_H\, Cz_{\alpha H}/Cz_{\alpha W}(1\text{-}d\varepsilon/d\alpha) \quad \text{(given in \% of CMA)}$$
- Considering **decoupled theory** results in making the phugoïd independent from the CG location.

Those hypothesis, that have proved worthwhile for full size aircraft at the preliminary design stage, prove to be defectuous in the case of sailplane model

2.1.2. Locally defined neutral point with respect to α

For sailplane model, the wing pitching moment is largely a non-linear function of alpha, **even in the middle of the usual flight range** (i.e. far from stall).
The **physical reason** behind this is the **movement of the laminar bubble** along an alpha polar, which produce a variation of pitching moment $C_m^{25}{}_W$
The phenomenon of massive laminar bubble being specific to medium Reynolds number, this aspect is not really preponderant for high Reynolds number, meaning for full size aircrafts.

This aspect can be taken into account by considering locally the neutral point $X^{\alpha}_{NP\,W}$ of the wing as a function of alpha :

$$\left.\begin{array}{c} C_m^{25}{}_W = C_m^{25}{}_W(\alpha) \\ \& \\ C_{LW} = C_{LW}(\alpha) \end{array}\right. \Rightarrow X^{\alpha}_{NP\,W}(\alpha,V0) = 0.25\text{-}(\partial C_m^{25}{}_{W}/\, \partial C_{LW})_{V0}(\alpha)$$

The position of the wing neutral point with regard to α for a given speed V0 is related to the slope of the curve $C_m^{25}{}_W(C_{LW})$ obtained through an alpha variation.

NB : Non linear effect on the tail, should be also evaluated :
- Through the behaviour of downwash ε, it can be considered as second order effects as soon as the sailplane is far from stall.
- Through the non linearity of the tail plane itself (typically dead band effect on the tail).

2.2. Wing neutral point with respect to V

<u>Definition :</u>
The neutral point with respect to V is the point were an increment (i.e. perturbation) of velocity V does not generate any pitching moment.

2.2.1. Usually admitted theory

It is usually admitted that the pitching moment and lift coefficients of a wing are not dependant of the velocity V. Similarly, it is admitted that the tail see the same velocity increment as the wing
At the end it means that any speed increment does not generate any pitching moment, and that neutral point with respect to V is not needed anyway.

2.2.2. Locally defined neutral point with respect to V

For sailplane model, the wing pitching moment is a (sometime non-linear) function of V, **even in the middle of the usual flight range**.

The **physical reason** behind this is the **evolution of the size of the laminar bubble with Reynolds number** when along an alpha polar, which produce a variation of pitching moment $C_m{}^{25}{}_W$
The phenomenon of massive laminar bubble being specific to medium Reynolds number, once again this aspect is not really preponderant for high Reynolds number, meaning for full size aircrafts.

This can be solved by considering locally the neutral point as a function of alpha

$$\left.\begin{array}{c} C_m{}^{25}{}_W = C_m{}^{25}{}_W(V) \\ \& \\ C_{LW} = C_{LW}(V) \end{array}\right. \Rightarrow X^V{}_{NP\,W}(\alpha 0,V)=0.25-(\partial C_m{}^{25}{}_{W/}\ \partial C_{LW})_{\alpha 0}\ (V)$$

The position of the wing neutral point with regard to V for a given incidence $\alpha 0$ is related to the slope of the curve $C_m{}^{25}{}_W(C_{LW})$ obtained through an V variation.

NB : Non linear effect on the tail should be also evaluated :
- Through the behaviour of downwash ε, but it can be considered as second or even 3rd order effects.
- Through the V-dependancy of the tailplane itself.

2.3. Combined perturbation ($d\alpha$,dV)

At the end, for any **combined perturbation** on incidence and speed ($d\alpha$,dV) (which is encountered in true life…), the pitching moment response is a mix of α and V effects.

The resulting pitching moment then created can be evaluated using the partial neutral point positions $X^{\alpha}_{NP\,W}$ and $X^{V}_{NP\,W}$:

$$dC_m^{25}{}_W(\alpha_0,V_0) = X^{\alpha}_{NP\,W}(\alpha_0,V_0)_W\,(\partial C_{L\,W/}\,\partial\alpha)_{V0}\,d\alpha + X^{V}_{NP\,W}(\alpha_0,V_0)\,(\partial C_{L\,W/}\,\partial V)_{\alpha 0}\,dV$$

As far as linearised dynamic study can be of interest for very non linear aerodynamics behaviour, the definition of the speed neutral can be integrated into the state matrix representative for the sailplane.

$$C_u(\alpha_0,V_0) = (\partial C_{L\,W/}\,\partial V)_{\alpha 0}$$
$$D_u(\alpha_0,V_0) = X^{V}_{NP\,W}(\alpha_0,V_0)\,(\partial C_{L\,W/}\,\partial V)_{\alpha 0}$$

$$\dot{X} = \mathbf{A}X$$

$$X = \begin{pmatrix} u \\ \gamma \\ \alpha \\ q \end{pmatrix} \quad , \quad \mathbf{A} = \begin{bmatrix} A_u & A_\gamma & A_\alpha \approx 0 & A_q \approx 0 \\ B_u & 0 & B_\alpha \approx 0 & B_q \approx 0 \\ C_u & 0 & C_\alpha & C_q \\ D_u & 0 & D_\alpha & D_q \end{bmatrix}$$

De-coupled phugoïd mode

De-coupled short period mode

New coupling terms for low Re dynamic analysis

This causes the apparition of coupling terms C_u & D_u associated with low Re aerodynamics.

This is a good illustration that the hypothesis of decoupling between phugoïd and short period mode is rather not representative of the dynamic behaviour of model of sailplane.

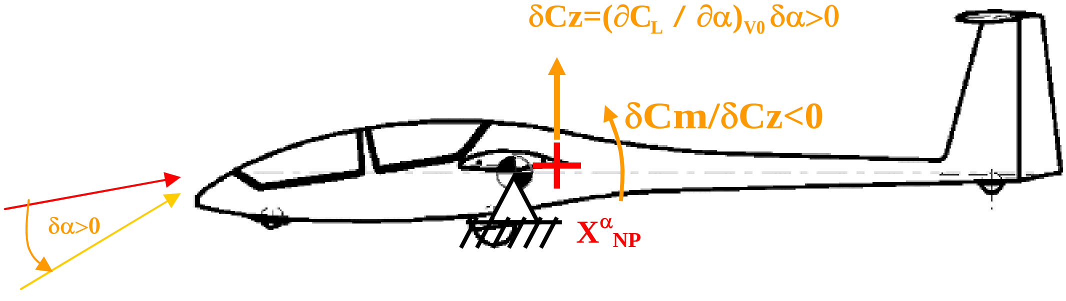
2.4. Practical use of partial neutral points

The interest of defining partial neutral point is to illustrate the **initial effect** of an α perturbation (resp. a V perturbation) in term of motion, and then be able to study the alpha stability (resp. speed stability) according to the CG location.

2.4.1. Positive α stability

A sailplane is α–stable if a small perturbation $\delta\alpha$>0 creates a pitch down moment (nose to the actual wind).

It means CG should be in front of X^{α}_{NP} for having a α–stable sailplane.

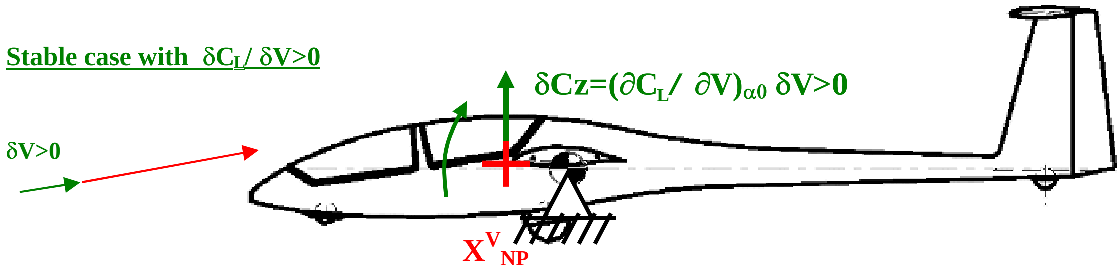


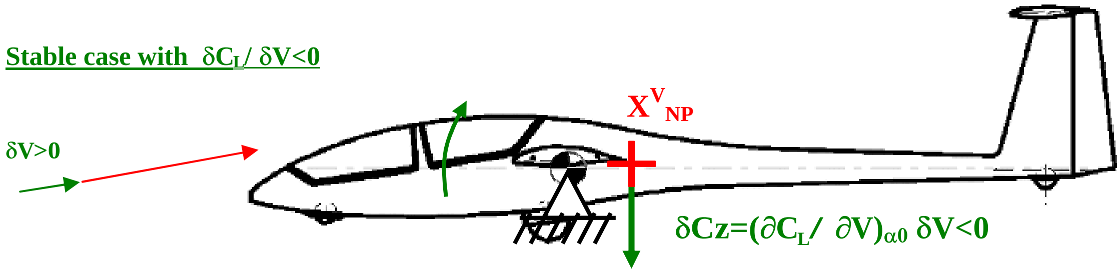
α-stability is the usual, by the book, notion of "static stability"

2.4.2. Positive V stability

A sailplane is V–stable if a small perturbation δV>0 creates a pitch up moment (acceleration should be transformed in height)

It means CG should be in front of X^{V}_{NP} for having a V-stable sailplane.





V-stability is a key point for phugoïd behaviour, i.e. so called "diving test" of modellers

2.5. Limitation of linearized theory

This may seem an evidence, but it must be kept in mind that linearization is a powerful tool providing the physic to be represented is reasonably linear. Then the use of

- Aerodynamic effects of alpha are reasonably linear over quite large part of the flight domain, meaning the neutral point with respect to alpha is rather easy to compute and to use for analysis.
- On the other hand, the effect of V on aerodynamic coefficient is largely non linear. The position of neutral point with respect to V is then very dependant upon the (α,V) parameters. Practically speaking, it is difficult to use this point for CG prediction, but it helps understanding the physical behaviour

3. Simulation of the non linear dynamic

The aerodynamic of the sailplane models being widely non linear, there are some point were linearized theory is not the best tool for understanding properly what is observed in flight.

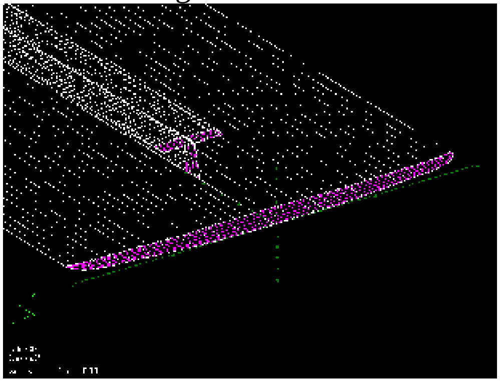
A simulator performing the integration for the non linear equation of airplane motion, using non linear modelling for the aerodynamic coefficient as function of both α and V, would be of great interest.

4. Practical study of Jade sailplane

4.1. Presentation of the Jade sailplane

The Jade sailplane aero-towed FAI sailplane, for the F3I class. It is a home made design.

Span : 3850mm
Area : 83.6dm²
Mass : 4550g



For more information :
http://perso.orange.fr/scherrer/matthieu/english/f3ie.html

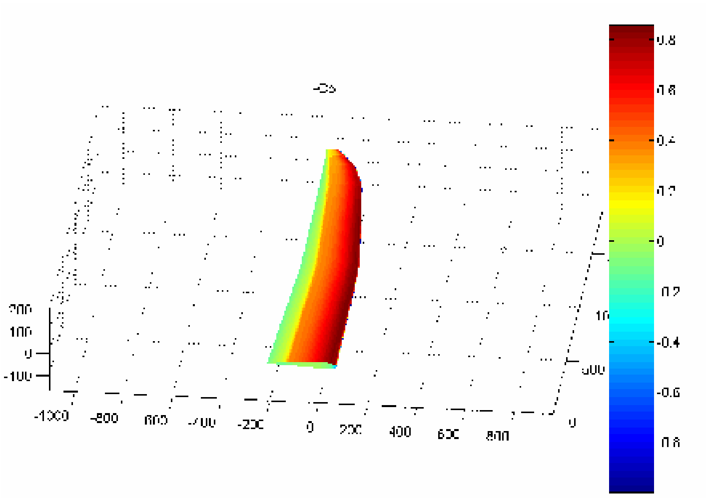
4.2. Low Reynolds aerodynamic characteristics

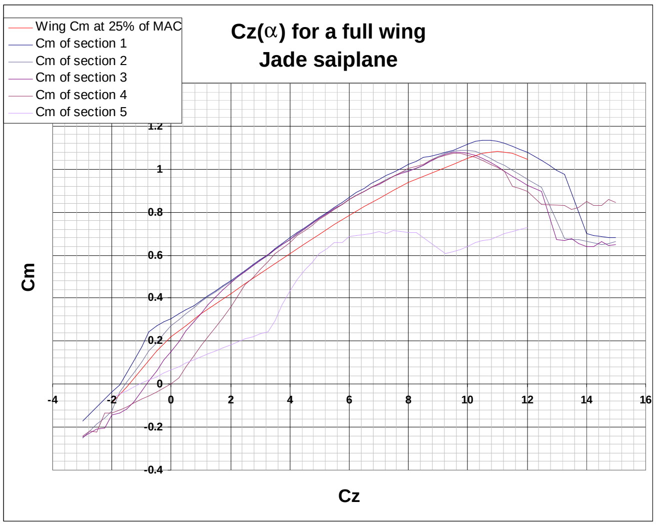
Without wind tunnel testing, the assessment of the actual pitching moment at 25% of the AMC of a wing, including all the non linear behaviour vs. α and V is a rather sensitive topic.

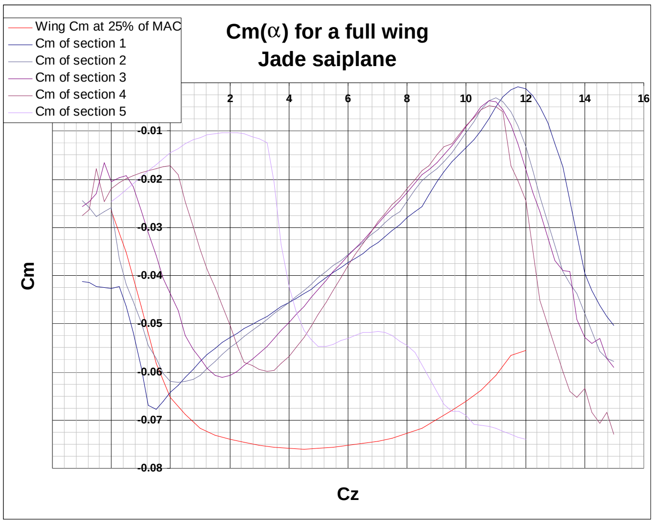
- Xfoil can reasonably well predict the pitching moment for an airfoil. Nevertheless, all the airfoils along a wing do not work in the same condition, causing the Cm to be different for each station of the span.
- On top of that, the moment induced by the sweep of the wing is to be taken into account and is very dependent to the spanwise lift distribution.

A tool that computes spanwise distribution of both C_L and C_m including non linear behaviour, is then needed.
For the following study, Miarex was used. This is a home-made lifting line code including determination of the circulation with non linear airfoil characteristics.
More info at :
http://perso.orange.fr/scherrer/matthieu/aero/miarex.html

NB : theory used by Miarex is not anymore valid as soon as sweep angle does affect significantly the lift distribution. Practically speaking, it means that it cannot cope with sweep angle over 10-15deg.







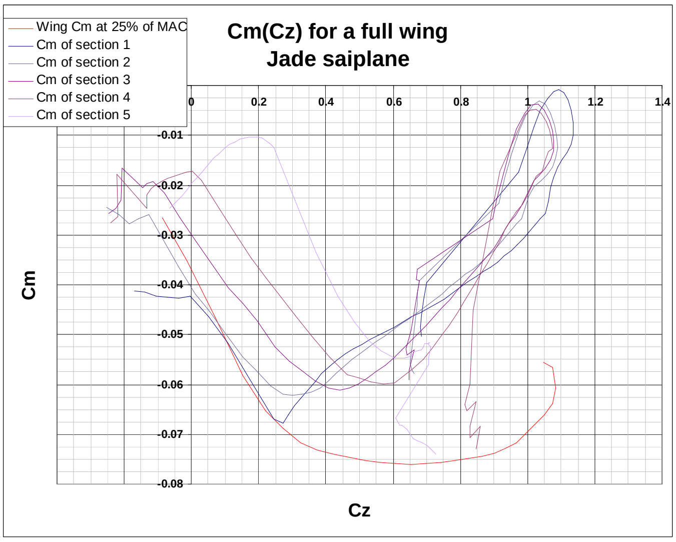
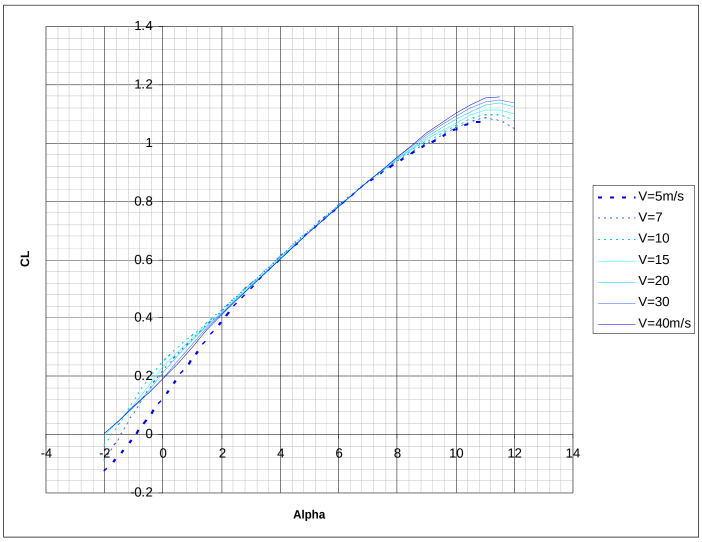


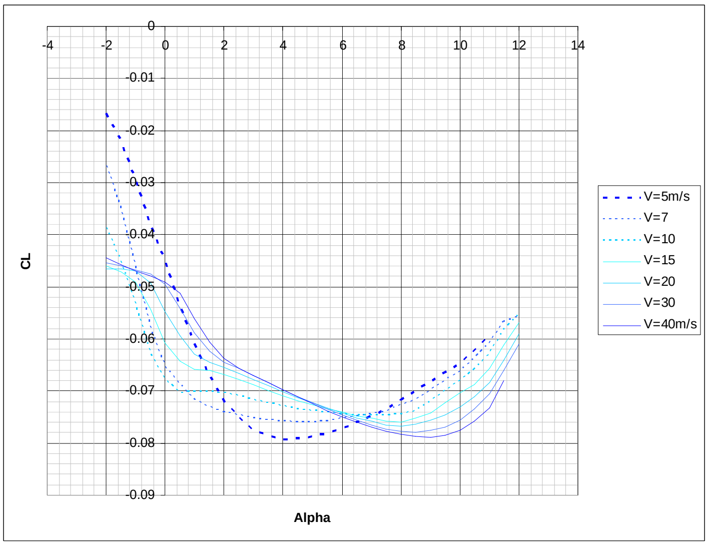
Illustration : Lift and moment of a wing are different from lift and moment of an airfoil.
C_L and C_m characteristics for the Jade wing at V=7m/s (C_m is given at 25% of AMC)
as computed by Miarex, using Xfoil for 2D section characteristics

NB : 2D data are a reasonable approximation for wing C_L, but not for C_m

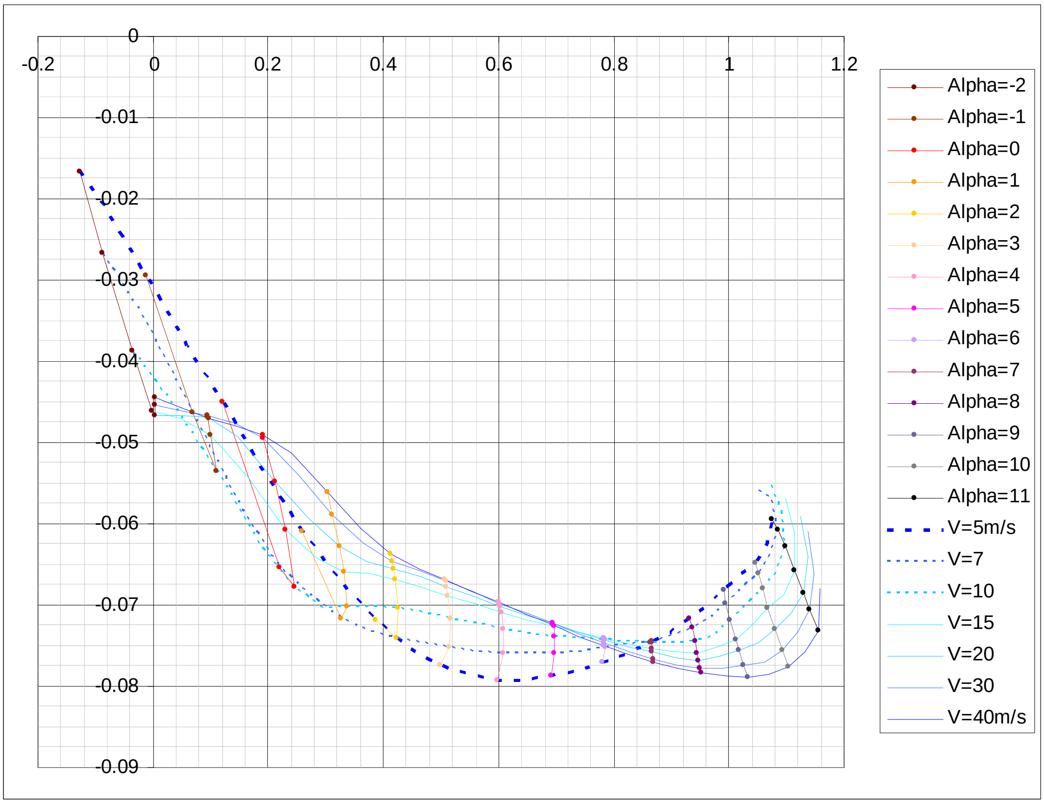
Low Reynolds aerodynamics characteristics of the Jade sailplane model
As computed from Miarex



C_L as function of α and V



C_m^{25} as function of α and V

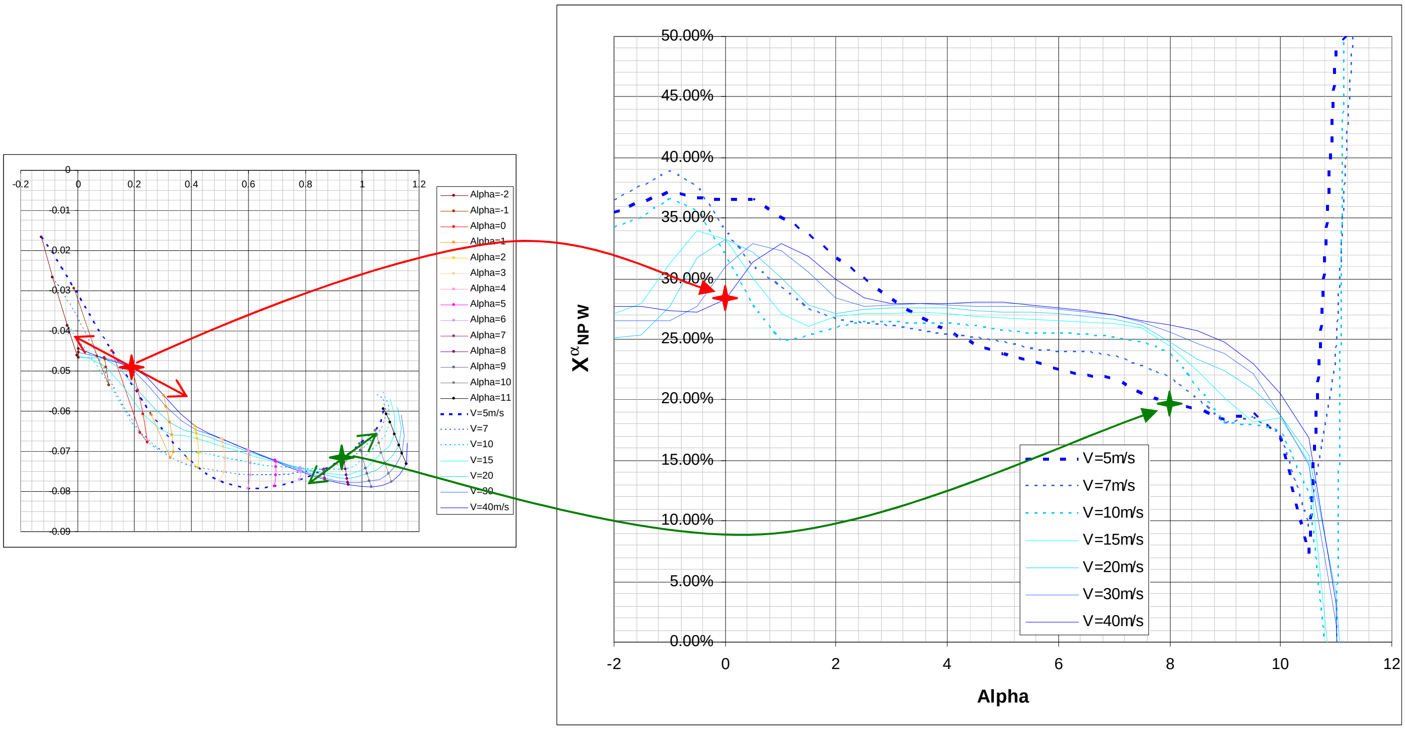


$C_m^{25}(C_L)$ curves for varying α and V

4.3. *Discussion on α-stability*

As in 2.1.2, the position of the neutral point with respect to α is related to the **slope of the $C_m(C_L)$ curve, along an α variation**. This should be computed for each point of the flight domain defined by (α0, V0)

Two example for the calculation of neutral point with respect to α are given, for (α=0, V=40m/s) in red and (α=8, V=5m/s) in green.



The inviscid value position of the neutral point is the famous 25% of AMC. Taking into account more detailed aerodynamic modelling, it can be seen that this point is located between 35% and 20% of the AMC within normal flight range (α in [-2, 10deg], V in [5-40m/s]). This is due to the effect of Reynolds number (varying with V) on a three-dimensional wing.

NB : This study deals only about **wing neutral point**. For concluding about the sailplane stability, **sailplane neutral point** is to be considered adding the effect of the tail on top of this. This effect of the tail can be considered as a constant shift (backward) dependant on tail volume, as a first approximation.

Then the evolution of $X^{\alpha}_{NP\ W}(\alpha_0,V_0)$ with α0 and V0 can be detailed in term of

Evolution of $X^{\alpha}_{NP\ W}$ with incidence

- At low angle of attack, a sudden movement of $X^{\alpha}_{NP\ W}$ is observed. This is linked to the evaluation of transition on the under side of the airfoil.
- For intermediate angle of attack, the evolution of $X^{\alpha}_{NP\ W}$ is more regular. For the highest speeds, this point is even more or less fixed (around 28% of AMC).
- When approaching the stall, it can be seen that $X^{\alpha}_{NP\ W}$ moves forward. This is likely to cause the neutral point of the sailplane to move forward to the CG location, hence creating unstable α behaviour when approaching stall.

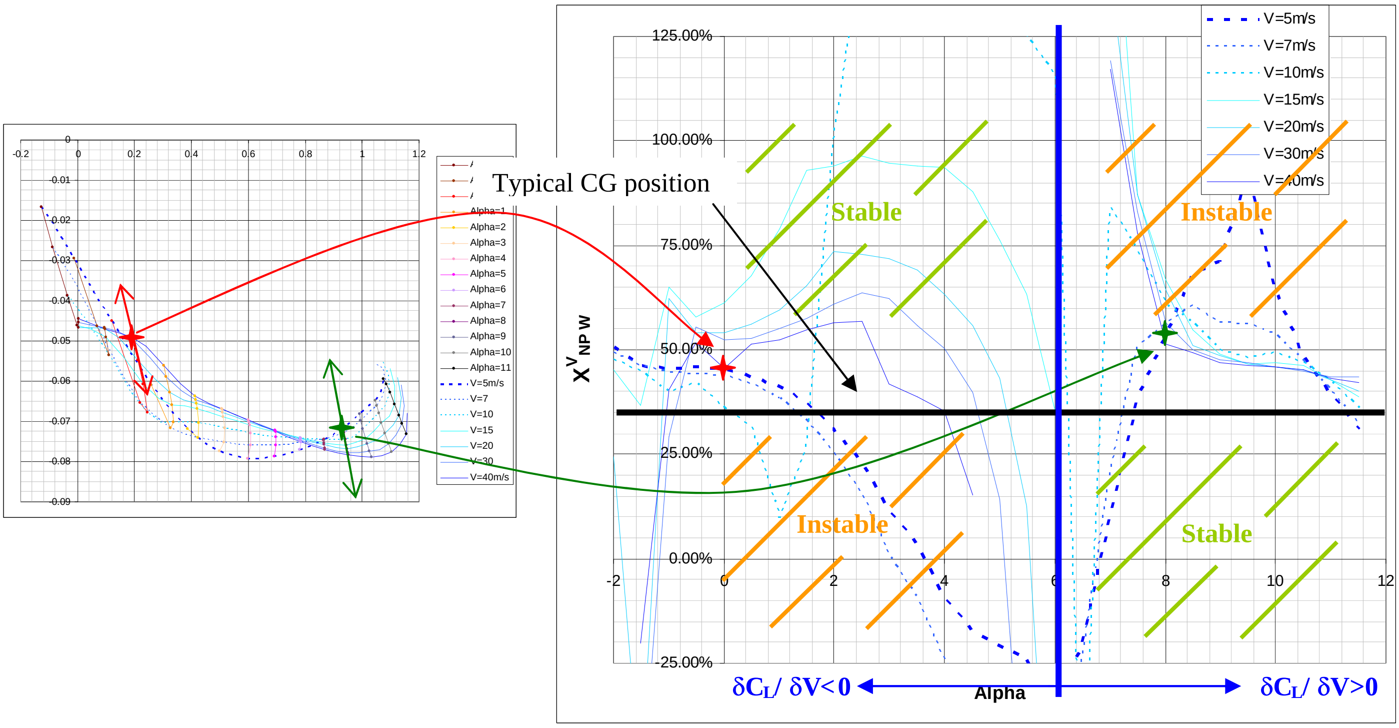
Evolution of $X^{\alpha}_{NP\,W}$ with velocity

- For medium alpha range, increasing V causes $X^{\alpha}_{NP\,W}$ to move backward, whereas for low alpha range, increasing V causes $X^{\alpha}_{NP\,W}$ to move forward. “Effective static margin” of the sailplane, taking into account the effect of tails, will be affected in a similar manner.
- The lower the speed, the more $X^{\alpha}_{NP\,W}$ evolves with α. At high Re number, situation about stability is simpler.

At the end, the movement of $X^{\alpha}_{NP\,W}$ justifies the necessity of static margin. Typical recommended value static margin of a sailplane model is commonly given between 8 and 15% of AMC.
Extend of $X^{\alpha}_{NP\,W}$ variation of position can exceed the provision for static margin. This is more and more the case when considering low Reynolds number, meaning slow & small model of sailplane.
Also, since detailed variation of $X^{\alpha}_{NP\,W}$ position is dependant on airfoil geometry, we have here an explanation for the commonly admitted dependency of CG setting to the airfoil.

4.4. Discussion on speed stability

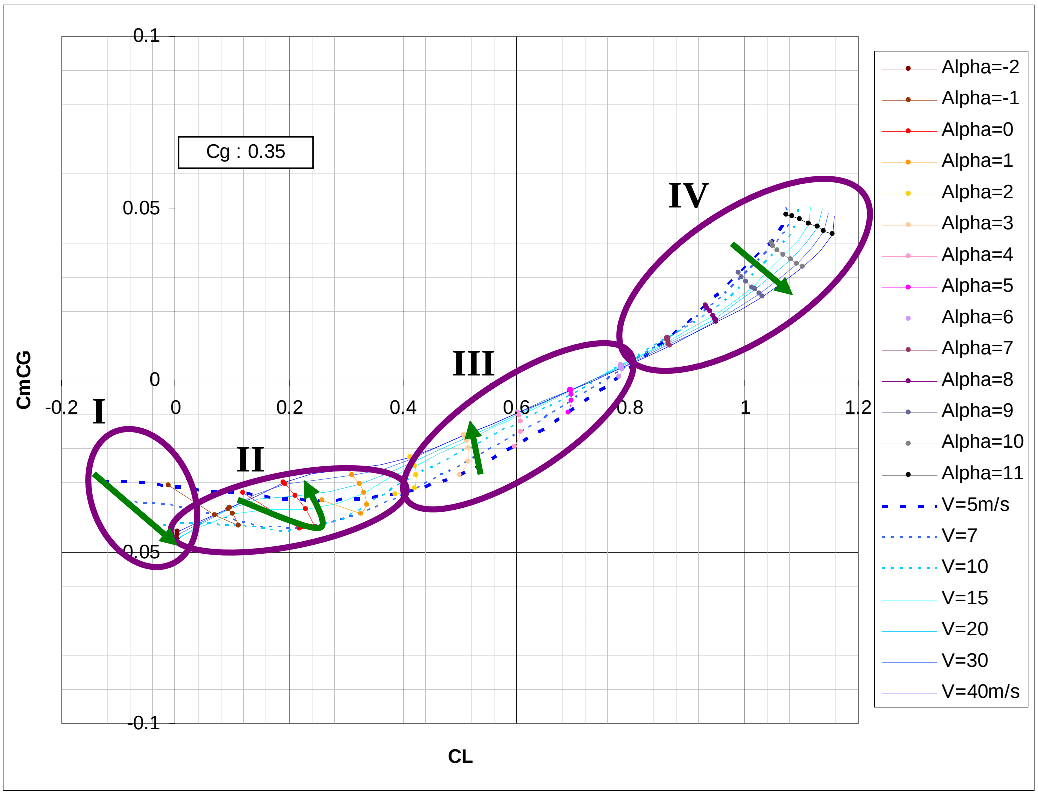
Similarly, as in 2.2.2, the position of the neutral point with respect to V is related to **the slope of the $C_m(C_L)$ curve, along an V variation**. This should be computed for each point of the flight domain defined by (α0, V0).
The same two example as before are given, for (α=0, V=40m/s) in red and (α=8, V=5m/s) in green.



The analysis of speed stability using the neutral point X^V_{NP} with respect to V is unusual and not straight forward. The main reason for that is that $\partial C_{LW}/\partial V$ can be either positive or negative (whereas for α-stability $\partial C_{LW}/\partial\alpha$ is always positive in usual flight domain). As a result the stable position of the CG relatively to X^V_{NP} is dependant over the sign of $\partial C_{LW}/\partial V$.

For the Jade sailplane X^V_{NP} does move a lot over flight domain. This is due to the fact that lift gradient with speed $\partial C_{LW}/\partial V$ is rather a small quantity, and is not linearly varying with alpha & speed. At the end, we are looking for a large lever arm since the lift force caused by a speed variation is rather small, whereas the effect on pitching moment is visible.

For an easier understanding, it is possible to read the speed stability characteristic directly from (C_L, $C_{m\,W}^{CG}$) mapping. The $C_{m\,W}^{CG}$ is the wing pitching moment taken **at the CG location**, this is the C_m to be considered for the influence of wing in longitudinal flight dynamics (the effect of tail being not taken into account here).



For a CG located at 35% of AMC, four main area can be determined :

- Area I : Very low C_L is unstable with speed
  As sailplane accelerates (from doted to full line) the wing does experiment a pitch down moment. A tendency to diving appears, causing the speed to be increased. This is the C_L area where the called "tuck under phenomenon (non linear pitch down motion at high speed) does occur.

- Area II : Low C_L is conditionally stable with speed
  Though at low speed the pitching gradient is the same as in area I, an inversion of the

trend is observed when speed increases. When have here speed instability at low speed, and stability at high speed

- Area III : Medium C_L is stable with speed

In the area of main interest for modellers, the wing of the jade is speed stable. Indeed,

- Area IV : High C_L is unstable with speed

When approaching the stall, the wing of the Jade becomes speed-instable. When decelerating, a pitch up motion is experienced

5. Conclusion

Starting point was an illustration of difficulties for understanding the dynamic behaviour of sailplane models. The prospect of this paper was to properly sort out what is needed for a better understanding of this aspect.

First specific aspect of low Re aerodynamic for sailplane model and their consequences on aerodynamic coefficient were pointed out. It is explained that the **aerodynamic** of sailplane models, and then their dynamic behaviour, **is widely dependant on speed**.
This is much less the case for full size sailplane and aircraft. This is probably why this aspect is scarcely discussed in the books.

Deriving classical linearization process to the specificity of low Re aerodynamic leads to better understanding of defect of classical "from the book" theory.
It is pointed out that :

- Position of actual neutral point with respect to α (hence actual "static margin") moves upon speed and incidence, leading to a **variable α-stability over flight range.**
- The concept of "speed stability" is derived from the analysis of the effect of speed variation at constant α. Practical use of neutral point with respect to V proved nevertheless not to be that easy.
- The interference of speed on pitching moment creates a much to the **failure of usual short period & phugoïde eigen-modes decoupling hypothesis** for sailplane model case.

A practical study of the Jade sailplane is then detailed. Without proper wind tunnel testing, it is shown that **advanced modelling of 3D wings is needed** for properly evaluate low Reynolds aerodynamic characteristic in pitching moment. Miarex tool was used for the study of this model.
Careful study of aerodynamic behaviour (C_L and C_m versus α and V) shows how intricate effect of α and V is on pitching moment at low Reynolds number.

At the end, it appears that a **proper simulator is needed** to gain in detailed understanding of sailplane model flight dynamic as observed in flight. Such a simulator would allow taking the full benefice of advance pitching moment modelling, and make a status about actual consequences of non linear effect on sailplane model motion.

Special thanks to François for his support,
and to Thierry for pushing me to think !