# A sailplane oriented way to write classical steady flight equations

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## Contents

While performing miscellaneous calculation on sailplanes aerodynamics and flight mechanics, I develop a simpler way to write and manipulate steady flight equations.

This leads to a really clear and easy to handle formalism, that prevent from doing mistake when performing tricky study. Moreover there is a physical meaning to this writing.

Nothing brand new, just something that helps going further into equation handling, optimization process and so on. . .

# 1 Classical steady flight equations

It is usual to write linearized equilibrium for steady flight as follows :

$$\begin{cases} 1/2\rho S V^2 C_L = mg\,cos(\gamma) \simeq mg \\ 1/2\rho S V^2 C_D = mg\,sin(\gamma) \simeq mg\,\gamma \end{cases}$$

From those expressions, velocity $V$ and sink speed $Vz$, that is the speed polar $(V, Vz)$, can be extracted as follows :

$$\begin{cases} V = \sqrt{\frac{2g}{\rho}\frac{m}{S}}\frac{1}{\sqrt{C_L}} \\ Vz = \sqrt{\frac{2g}{\rho}\frac{m}{S}}\frac{C_D}{{C_L}^{3/2}} \end{cases} \tag{1}$$

# 2 New way to write steady flight equations

## 2.1 The new equation set :

I propose to consider the following reference velocity :

$$V_1 = \sqrt{\frac{2g}{\rho}\frac{m}{S}} \tag{2}$$

This corresponds to the velocity V obtained in Eq. 1, with $C_L = 1$.
This reference speed is a function of air properties, through its density $\rho$ and sailplane properties, through its mass $m$ and it area $S$, or his wing loading $\frac{m}{S}$.

Thanks to this set of equation, it is then possible to write the classical steady flight equation as follows :

$$\begin{cases} V = V_1 \dfrac{1}{\sqrt{C_L}} \\ Vz = V_1 \dfrac{C_D}{{C_L}^{3/2}} \end{cases} \tag{3}$$

### 2.2 Why using another way of writing those equations ?

Using classical expressions for Vz and V mixes a lot of variables, than may leads to mistake when performing symbolic calculation. Clarity is one reason for writing the equation this way.

Moreover, there are physical reasons to write the equation as proposed : on one side you have aerodynamics through $C_L$ and $C_D$, whereas on the other side you have sailplane characteristics through $m$ and $S$ and air property through $\rho$.
Separating the different effect on the speed polar $(V, Vz)$ is another reason to write the equations as follows.

### 2.3 Steady turn case

For steady turn case, the former equation set can be easily adapted :

$$\begin{cases} V = V_1 \dfrac{1}{\sqrt{C_L\, cos(\phi)}} \\ Vz = V_1 \dfrac{C_D}{(C_L\, cos(\phi))^{3/2}} \end{cases} \tag{4}$$

Where $\phi$ is the bank angle.

### 2.4 Derivation of the equations for conceptual use

It can be interesting to a have the expression of the gradient of $V_1$ for different variable. Those derivatives can be used in conceptual optimization process.

$$\left\{\begin{aligned} \frac{\partial V_1}{\sqrt{\partial h}} &= -\frac{1}{2\rho}\frac{d\rho}{dh}V_1 \\ \frac{\partial V_1}{\sqrt{\partial m/S}} &= \frac{1}{2m/S}V_1 \\ \frac{\partial V_1}{\sqrt{\partial m}} &= \frac{1}{2m}V_1 \\ \frac{\partial V_1}{\sqrt{\partial S}} &= -\frac{1}{2S}V_1 \end{aligned}\right. \tag{5}$$

A example of the use of such derivative is given at § 3.4

## Summary

The main things to keep in mind :

◇ Steady flight equation can be written as :

$$\left\{\begin{aligned} V_1 &= \sqrt{\frac{2g}{\rho}\frac{m}{S}} \\ V &= V_1\frac{1}{\sqrt{C_L}} \\ Vz &= V_1\frac{C_D}{{C_L}^{3/2}} \end{aligned}\right.$$

◇ $V_1$ embodies the effect of sailplane geometry and mass.

◇ $C_D$ and $C_L$ represent the effect of glider aerodynamics.

# 3 Some examples

## 3.1 Computing variable Reynolds polars

Xfoil code from M. Drela proposes variable Reynolds drag polar calculation. That is for any $C_L$ value, Reynolds number is adapted, and computed with the velocity given by the equilibrium equation Eq. 1.

Parameter $\Re e\sqrt{C_L}$ is to be set as a input of computation, and can be written as a function of $V_1$ only. Indeed :

$$\Re e\sqrt{C_L} = \frac{lV}{\nu}\sqrt{C_L} = \frac{l\frac{V}{\sqrt{C_L}}}{\nu}\sqrt{C_L}$$

That is :

$$\Re e\sqrt{C_L} = \frac{lV_1}{\nu} \tag{6}$$

## 3.2 Dimensionless speed polar for a sailplane

If the sailplane is given, its aerodynamics is given, and then the ratio $\frac{C_D}{{C_L}^{3/2}}$. It is then easy to define an universel, dimensionless speed polar for this glider :

$$\begin{cases} \frac{V}{V_1} = \frac{1}{\sqrt{C_L}} \\ \frac{Vz}{V_1} = \frac{C_D}{{C_L}^{3/2}} \end{cases} \tag{7}$$

Real speed polar is only directed by $V_1$ value. One can then compute $V_1$ according to ballasted or un-ballasted condition, or altitude for instance.

## 3.3 Inverting the speed polar of an existing sailplane

For a given existing sailplane, it is possible to get speed polars from commercial documentation. Inverting those speed polars to get the aerodynamic polar can be interesting. Using $V_1$ really make the process clear :

$$\begin{cases} C_L = \left(\frac{V_1}{V}\right)^2 \\ C_D = \frac{V_1^2 Vz}{V^3} \end{cases} \tag{8}$$

Here is an example for ASW 28. For a wing loading of 38.1kg/m$^2$ in standard condition, reference speed is equal to $V_1 = 24.7m/s$.

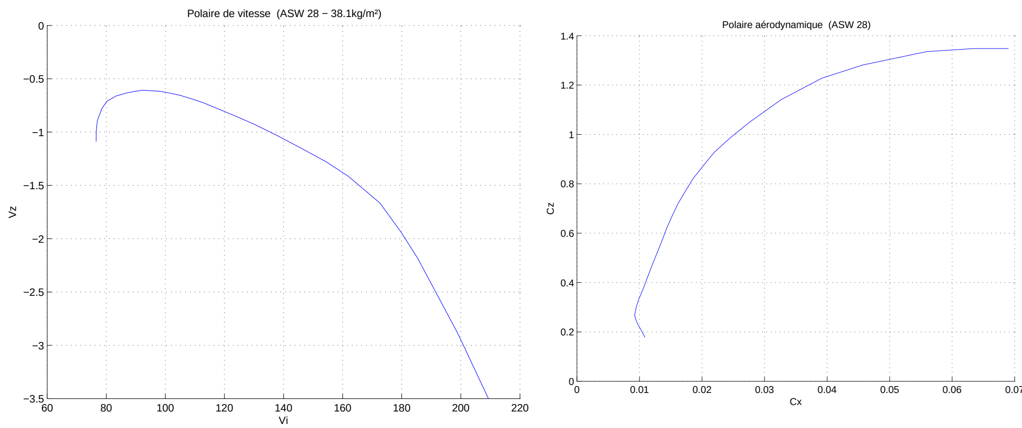


Figure 1: Available speed polars for ASW 28 sailplane ($38.1\text{kg/m}^2$) and corresponding computed aerodynamic polar

The inversion formulas allow to associate the polar speed point ($V = 130km/h$, $Vz = -0.92m/s$) to the aerodynamic polar point ($C_L = 0.4707$, $C_D = 0.0119$). Processing each speed polar points gives each aerodynamic polar points.

## 3.4 Theoretical effect of wing aspect ratio on sink speed

Here is an example of what this writing of the steady flight equilibrium can bring to the process of equation manipulation.

If you want to explicit $\dfrac{\partial Vz}{\partial \lambda}$, the gradient of sink speed in respect to AR $\lambda$, classical set of equation is very heavy to handle. Handling $V_1$ is really interesting in clarifying the symbolic calculation. The derivatives given in § 3.4 will be of great use !

Lets describes the symbolic calculation process :
We will at first split the gradient according to derivation of aerodynamics effect and derivation of sailplane geometry effect.

$$\frac{\partial Vz}{\partial \lambda} = \frac{\partial V_1 \frac{C_D}{{C_L}^{3/2}}}{\partial \lambda} = \frac{\partial V_1}{\partial \lambda}\frac{C_D}{{C_L}^{3/2}} + V_1 \frac{\partial \frac{C_D}{{C_L}^{3/2}}}{\partial \lambda} \tag{9}$$

Dependency of aerodynamics ratio $\dfrac{C_D}{{C_L}^{3/2}}$ to $\lambda$ (right terme in eq. 9) will be first detailed.

Considering drag as : $\dfrac{C_D}{{C_L}^{3/2}} = \dfrac{C_{D0}}{{C_L}^{3/2}} + \dfrac{C_{D airfoil}}{{C_L}^{3/2}} + \dfrac{1}{\pi e \lambda}\sqrt{C_L}$, we get :

$$\frac{\partial \frac{C_D}{{C_L}^{3/2}}}{\partial \lambda} = \frac{\partial C_{D airfoil}}{\partial \Re e}\frac{\partial \Re e}{\partial \lambda}\frac{1}{{C_L}^{3/2}} - \frac{1}{\pi e \lambda^2}\sqrt{C_L}$$

Then dependency of $V_1$ to $\lambda$ (left terme in eq. 9) is to be detailed :

$$\begin{aligned}\frac{\partial V_1}{\partial \lambda} &= \frac{\partial V_1}{\partial m}\frac{\partial m}{\partial \lambda} + \frac{\partial V_1}{\partial S}\frac{\partial S}{\partial \lambda} \\ &= \frac{V_1}{2}\left(\frac{1}{m}\frac{\partial m}{\partial \lambda} - \frac{1}{S}\frac{\partial S}{\partial \lambda}\right)\end{aligned}$$

Where $\frac{\partial m}{\partial \lambda}$ is the dependency of the mass to the aspect ratio : the higher the aspect ratio, the heavier the wing for a given stiffness. Since $\frac{\partial S}{\partial \lambda} = -\frac{b^2}{\lambda^2}$, we get :

$$\frac{\partial V_1}{\partial \lambda} = \frac{V_1}{2}\left(\frac{1}{m}\frac{\partial m}{\partial \lambda} + \frac{1}{S}\frac{b^2}{\lambda^2}\right) = \frac{V_1}{2}\left(\frac{1}{m}\frac{\partial m}{\partial \lambda} + \frac{1}{\lambda}\right)$$

At the end, neglecting the effect of AR on Reynolds number (that is $\frac{\partial \Re e}{\partial \lambda} \simeq 0$), if we bring all this together we get :

$$\frac{\partial Vz}{\partial \lambda} = V_1\left(\frac{1}{2}\left(\frac{1}{m}\frac{\partial m}{\partial \lambda} + \frac{1}{\lambda}\right)\frac{C_D}{{C_L}^{3/2}} + \frac{1}{\pi e \lambda^2}\sqrt{C_L}\right) \tag{10}$$

Such a handling of symbolic expressions, still not simple, is much more easy with the $V_1$ reference speed.

# 4 Conclusion

The contents of this paper may appear just evident, but this way to write and handle steady flight equations helps a lot .It can be of very simple use, see § 3.1 or § 3.2. It also brings a lot of clarity to tricky calculation such as seen in § 3.4.

So, nothing brand new, but a precious help to symbolic calculation…