Total induced drag of a trimmed sailplane

matthieu.scherrer@free.fr

Influence of trimming on drag is not a straightforward subject.

It is difficult to know if the magnitude of this drag component is that important. The role played by different conception parameter and CG location is not an easy topic either.

The objective of this theoretical paper is to write equations that will help understand how the load sharing between wing and tail influence the total drag.
Pitching moment of the wing and geometrical parameters of wing and horizontal tail plane will be considered as design parameters. CG location will be a degree of freedom, set through theoretical static margin.

Symbols

Geometrical parameters :

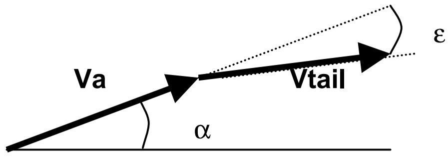
l	Reference chord length
S	Reference area (wing)
S_H	Horizontal tail plane area
λ_w	Wing aspect ratio
λ_H	Horizontal tail plane aspect ratio
A	Wing alone aerodynamic centre location
H	Horizontal tailplane alone aerodynamic centre location
F	Resulting sailplane aerodynamic centre location
$\sigma = \frac{GF}{l}$	Static margin
$V_H = \frac{S_H}{S}\frac{AH}{l}$	Horizontal tail plane volume

Aerodynamic coefficients :

Cz_w	Wing lift coefficient (in respect to S)
Cm_0	Wing lift coefficient (in respect to S)
Cz_H	Horizontal tail plane lift coefficient (in respect to SH)
$Cz_{A/C}=Cz_w+S/S_H.Cz_H$	Saiplane lift coefficient (in respect to S)

Speed composition at tail location

The horizontal tail sees the airflow deflected by the wing. Deflection is a small angle called ε.

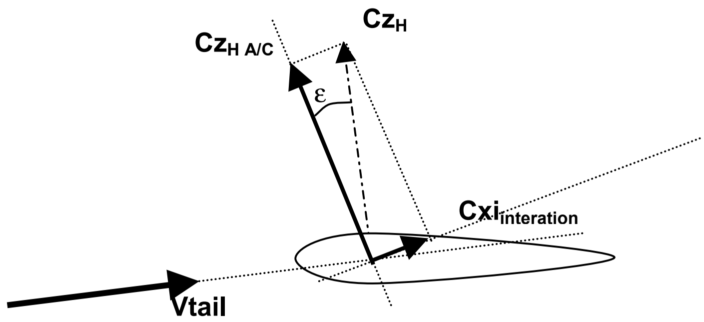


$$\varepsilon = \frac{2}{\pi\lambda_w} Cz_w$$

$$\varepsilon \text{ is small: } \sin\varepsilon \approx \varepsilon,\ \cos\varepsilon \approx 1$$

From its lift polar, one can know the lift generated by the horizontal tail plane, perpendicular the local speed axis **Vtail**. For considering aerodynamic coefficient for the sailplane, you have to consider **Va** axis for drag, and its perpendicular for lift.

So that considering the lift of the tail plane Cz_H, you have to split it to get the aerodynamic coefficients for the sailplane.
The result is a lift component $Cz_{H\ A/C}$ and a drag component $Cxi_{interaction}$.



NB : $Cxi_{interaction}$ can be ***negative***, that is it can represents a thrust component, when Cz_H is oriented downward.

Total induced drag

The total induced drag is the sum of the wing induced drag, the own induced drag of tail plane, plus the interaction term defined sooner :

$$Cxi_{tot} = Cxi_{wing} + Cxi_{interaction} + Cxi_H$$

The detail is :

$$Cxi_{tot} = \frac{1}{\pi\lambda_w} Cz_w^2 + \frac{S_H}{S}.\sin\varepsilon.Cz_H + \frac{S_H}{S}\frac{1}{\pi\lambda_H} Cz_H^2$$

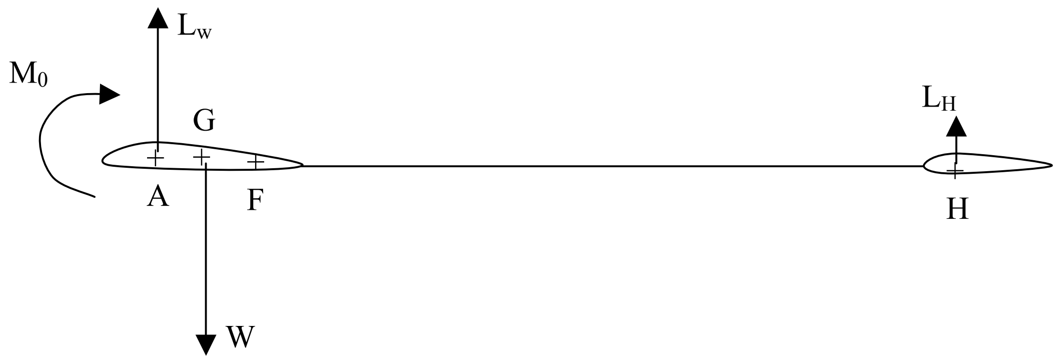
$$\text{with} \quad \sin\varepsilon \approx \varepsilon = \frac{2}{\pi\lambda_w} Cz_w$$

All in all, we get the following expression :

$$\boxed{Cxi_{tot} = \frac{1}{\pi\lambda_w}\left(Cz_w^2 + \frac{2S_H}{S} Cz_H.Cz_w + \frac{S_H}{S}\frac{\lambda_w}{\lambda_H} Cz_H^2 \right)}$$

Static trimming equation

For discussing trimming equation, here is a sketch :



A is the wing alone aerodynamic centre, H the tail plane alone aerodynamic centre, and F the resulting sailplane aerodynamic centre.

We have to translate trimming constraint.
In term of force, lift of the wing Lw plus lift of the tail plane LH must compensate the weight W of the sailplane.
In term of moment at F location, nose up component (wing lift pitching moment) must compensate all the nose down component (wing pitching moment, weight pitching moment and tail lift pitching moment).

We get :

$$L_w + L_H = W$$
$$M_0 + AF.L_w = GF.W + FH.L_H$$

This can be reduced to one single relationship describing lift sharing between wing and tail, using aerodynamic coefficients and non dimensional parameters :

$$Cm_0 + \left(\frac{AF}{l} - \sigma\right).Cz_w = \left[V_H - \frac{S_H}{S}\left(\frac{AF}{l} - \sigma\right)\right].Cz_H$$

If we suppose AF distance is negligible compared to AH distance, with classical VH value (less than 1), we can say:

$$V_H - \frac{S_H}{S}\left(\frac{AF}{l} - \sigma\right) = VH\left(1 - \frac{\frac{AF}{l} - \sigma}{\frac{AH}{l}}\right) \approx V_H$$

NB : this assumption is very accurate for classical sailplane configuration ($AF/l=0.5$, $\sigma= 0.15$, $AH/l \approx 0.375\lambda_w$)

We get as a final result the tail lift coefficient, as a function of wing lift coefficient and CG location (s parameter) :

$$\boxed{Cz_H \approx \frac{1}{V_H}\left[Cm_0 + \left(\frac{AF}{l} - \sigma\right).Cz_w\right]}$$

NB : The sailplane aerodynamic centre F is also a function of wing and tail geometry.

As a first guess : $\frac{AF}{l} = V_H \frac{1 + 2/\lambda_w}{1 + 2/\lambda_H}$

Total induced drag with trimming constraint, as function of Cz_w

So we have got two expressions, quite uneasy to handle. We will rewrite them in order to perform the calculation of total drag under trimming constraint, as follows :

- $Cxi_{tot} = \frac{1}{\pi\lambda_w}\left(Cz_w^2 + aCz_H.Cz_w + bCz_H^2\right)$ with : $a = \frac{2S_H}{S}$ and $b = \frac{S_H}{S}\frac{\lambda_w}{\lambda_H}$
- $Cz_H = \alpha Cz_w + \beta Cm_0$ with : $\alpha \approx \frac{\frac{AF}{l} - \sigma}{V_H}$ and $\beta \approx \frac{1}{V_H}$

By mixing those two equations together, we can expressed the so called "total drag under trimming constraint", as a function of wing aerodynamic coefficient Cz_w and Cm_0 :

$$\boxed{Cxi_{tot} = Cxi_{wing} + \Delta Cxi_{trim} = \frac{1}{\pi\lambda_w}\left(Cz_w^2\left(1 + a\alpha + b\alpha^2\right) + Cm_0.Cz_w\left(a\beta + 2b\alpha\beta\right) + Cm_0^2.b\beta^2\right)}$$

Well, this is not a so funny expression...
This is a polynomial expression in respect to Cz_w, just like usual quadratic drag. It is also a quadratic expression in respect to wing pitching moment coefficient Cm_0.
We will try to give a physical meaning to the different terms we can observe.

Term	Typical relative magnitude	Explanation
$a\alpha + b\alpha^2 = \frac{S_H}{S}\left[2.\frac{\left(\frac{AF}{l} - \sigma\right)}{V_H} + \frac{\lambda_w}{\lambda_H}\frac{\left(\frac{AF}{l} - \sigma\right)^2}{V_H^2}\right]$	0.0465	This is the equivalent loss of span efficiency caused by the trimming. It is a function of the ratio between CG position and Tail volume.
$\left(a\beta + 2b\alpha\beta\right)Cm_0 = \frac{2S_H}{S}\left(\frac{1}{V_H} + \frac{\lambda_w}{\lambda_H}\frac{S_H}{S}\frac{\left(\frac{AF}{l} - \sigma\right)}{V_H^2}\right)Cm_0$	-0.0545	This term is the gradient of drag due to wing lift. It originates at the tail location, were the deflection rotate the tail local lift. Then it can be negative. It is a function of CG position and Tail volume.
$b\beta^2 Cm_0^2 = \frac{S_H}{S}\frac{\lambda_w}{\lambda_H}\frac{1}{V_H^2}Cm_0^2$	0.0111	This is the direct impact of the drag increment caused by the pitching moment. It originates in the tail loading due to Cm0. It is a function of the tail volume and the ratio between surface AR. Divided by $\pi\lambda_w$, it corresponds to a drag value.

NB : Typical magnitude of those term were computed for a typical sailplane (λ_w=22, λ_H=6, V_H=0.4, AF/l=0.27, S_H/S=0.05, Cm0=-0.1)
All in all, comparing the final magnitude to the wing contribution, we get that trim drag is a small componet of a typical glider drag polar.

$$Cxi_{wing} = 0.014469Cz_w^2$$

$$\Delta Cxi_{trim} = 0.000673Cz_w^2 - 0.00075Cz_w + 0.000161$$

While considering this expression, you have to keep it mind that drag is given as a function of wing lift coefficient. For a given wing lift coefficient, load on the lift will change the sailplane total lift coefficient $Cz_{A/C}$, and then its speed.

Total induced drag with trimming constraint, as function of $Cz_{A/C}$

For minimizing the global drag at a given speed, one must consider the drag as a function of aircraft lift coefficient $Cz_{A/C}$

Sailplane lift coefficient is defined as follows

$$Cz_{A/C} = Cz_w + \frac{a}{2} Cz_H = \left(1 + \frac{a\alpha}{2}\right) Cz_w + \frac{a\beta}{2} Cm_0$$

So that

$$Cz_w = ACz_{A/C} + BCm_0$$

Using this, drag can be expressed as a new quadratic expression in respect to Cm_0 and $Cz_{A/C}$.

I am in the process of sorting all the term, in order to have a handable expression.

The final objective then will be to study optimal tail geometry and CG location in term of total drag under trimming constraint, at a given $Cz_{A/C}$.